Bertrand Competition with Sunk Cost

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June 2, 2017

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Abstract

This paper constructs a model that firms compete as Bertrand but they put some efforts on pre-production period with zero marginal cost while production. Each firm's effort cost is privately known. We show that if firms cannot observe the quantities the other firms made when they are setting price, low-cost firms will set a price more aggressively and high-cost firms will set a price higher than the firms in the market that production after naming the price. Moreover, a market with more elastic demand amplifies this effect. When the observation of quantity is possible, the market equilibrium will be consistent with the Cournot outcome, though firms compete for market share in Bertrand game. This is a similar result to Kreps and Scheinkman (1983) and Lepore (2008).

Keywords: seal bid auction, revenue equivalence, sunk cost,

JEL classification :D21, D24, D43, D44

1 Introduction

This chapter investigates Bertrand competition when firms produce in advance. In a standard Bertrand model, firms compete in terms of price and the winner takes all. The losing firms bear no loss in the bidding games. It can be viewed as all firms are sitting in a room and write down their sales price, then the firm with the lowest price wins and starts to produce the goods. In practice, a retailer orders the goods he needs before the sale season begins and pays for that order either before or after the goods were sold. Furthermore, an integrated firm who produces and sells the good by himself is more likely to bear the production cost before the market price is determined. Thus, production before a sale is an issue when the supply chain is taken into account.

We consider a Bertrand model with known market demand, where firms produce a certain amount they want before they sells it on the market. Also, each firm has privately known production cost. In this production in advance model with Bertrand competition, there are two seasons respectively, namely producing season and sales season. We assume no firms knows the quantities that other firms have produced, which transforms a Bertrand game with unknown cost as a first-price seller's auction into an all-pay seller's auction. Some of the properties of all-pay auctions have been applied in this study.

There are reasons for considering production in advance model. Financial frictions make part of the inventories to be unable to be backed up by the collateral or reputation of the retailer. The retailers might have either too many or too less inventories due to demand uncertainty. It is the literature of supply chain that motivates us to review the concept of inventory. In the supply chain literature, a standard model called "Newsvendor model" was introduced. The downstream firms, the newsvendors, sell final products with stochastic demand. Firms in the supply chain determine how many products to be produced whilst bearing the cost of possibly unsold good. Quote Chen [2] about newsvendor model:

> The model assumes complete certainty on the supply side, where, typically, an unlimited quantity can be produced at an exogenously given per-unit wholesale price. In reality, however, most industrial buyers face multiple potential suppliers with private information about their production costs.

Chen[2] notes that a procurement contract is optimal when it adopts the following procedures. First, the buyer proposes a price menu for procured products and delegates the quantity decisions to the winning supplier. The auction with an up-front lump-sum fee fits well in practice, like in slot allowance and vendor-managed inventory. The supply chain auction takes the uncertainty about suppliers' production cost into account. Meanwhile the wholesale price in the auction endogenously induces an "unlimited" but pre-determined quantity of supply. We provide another scenario in which the production not only is limited by the supply contract and technology but also takes place before the trade is made.

Auction theory and modern microeconomics, both theoretically and practically, grows fruitful insight on the process of price formation.[1] Two papers have provided

[1] See for examples in the textbook of Krishina[6] and Milgrom[8]

basic results for first-price and second-price auctions. Hansen [4] studies theoretical models of auction when the quantities of auctioned objects are not fixed. He constructs a model with negatively sloped demand and compares a first-price and a second-price auction in order to explain the process of price-formation. Hansen [4] concludes that while the open auction, which is equivalent to the second-price auction, has the same equilibrium bidding price between fixed and variate demand, whilst the seal-bid first-price auction gets a lower bid under negative demand setting than fixed demand. Spulber [10] proposes a Bertrand competition model with unknown rival's cost. In that paper, he asks the question whether the marginal-cost pricing is still hold when the cost is privately known by the seller itself. The result is similar to the one in Hansen [4] in first-price auction, illustrating equilibrium price is above the marginal cost for every type of firms and all firms bid a positive price implying positive expected profits.

Fibich et al.[3] study All-pay auctions with risk averse buyers. They show that buyers with low value will bid less than they will do in first-price auctions. On the other hand, buyers with high value will bid more in all-pay auctions than in first-price auctions. The risk averse behavior affects the optimal behavior of buyers to trade off between preventing losing the auction and losing the money they have bid.

We study all-pay seller's auction in which a firm with concave profit function is similar to a buyer with risk averse preference. The model shows a similar result to Fibich et al.[3] but based on a reverse intuition. The low-cost firm will set a more aggressive price while the high-cost firm will set a more conservative price compared to the first price seller's auction. The firm who learned a low cost is more likely to win and expects to prevent losing by lowering his price. Also, the firm who learned a high cost is more likely to avoid loss and produce less to meet the market demand by a high price when he wins. Both behaviors of the firms are the reverse version of risk averse bidders.

Moreover, we investigate that if firms observe their rivals' quantities of supply when they are setting price. It is the case like Kreps and Scheinkman[5] in public known cost and Leopore [7] in privately known cost. Both studies show that the Cournot outcome is an equilibrium in pricing competition. We found that, unlike their complex rationing rule for competition, the winner-take-all rationing rule still get the Cournot outcome with privately known cost. This is an approach that is more close to the original Bertrand competition.

Next, we develop a production in advance models and describe the symmetric equilibrium properties in a duopoly case. Then an extension that firms observe rival's quantity supplied is discussed. Concluding remarks provide the welfare and policy implications.

2 Produced in advance models

We construct a model to illustrate the scenario that firms announce their prices when they have delivered their product onto the marketplace. This is a proxy of the process that firms should get their products ready before consumers go into the mall. Under traditional Bertrand competition, the winner-take-all rationing rule enables only the firm who sets the lowest price produce the good and serves the market.

When the products are produced in advance, all of the firms bear certain production cost no matter who serves the market. Then the Bertrand game becomes an all-pay like auction.[2] Specifically, each firm produces the goods in advance with the quantity according to a known market demand. We denote this scenario as game $\Gamma(A)$. Also the traditional Bertrand game when rival's cost is unknown is denoted by game $\Gamma(1st)$.

Suppose n firms compete in a winner-take-all market. Assume a general cost function $c(q_i, \theta_i)$ is a function of quantity q_i and the cost parameter θ_i for firm $i \in \{1, 2, ..., n\}$. Suppose cost is increasing and convex in q_i, i.e., $c_1 \geq 0$ and $c_{11} \geq 0$, and the marginal cost is increasing in θ, i.e., $c_{12} \geq 0$. Each firm's parameter of cost is drawn from a common prior $\theta_i \in [\underline{\theta}, \overline{\theta}]$ with distribution function $F(\theta_i)$ and $F'(\theta_i) = f(\theta_i)$. The market demand is $D(p)$ where p is the smallest bid of all firms. The demand is twice continuously differentiable function and $D'(p) \leq 0$.

First, note that when $n = 1$, the monopoly case, firm in game $\Gamma(A)$ maximizes the same profit function with the monopoly pricing in game $\Gamma(1st)$:

$$\pi(p, \theta) = pD(p) - c(D(p), \theta). \tag{1}$$

Denote the optimal monopoly price as $p^m(\theta)$ and quantity as q^m, where q^m satisfies $q^m = D(p^m)$. The first order condition of this problem is

$$\frac{p^m - c_1}{p^m} = \frac{1}{\epsilon_{p^m}}, \tag{2}$$

[2] Notice that in an all-pay auction buyers name the price and all of them pay the prices to the seller. Here, the other side of the pricing game, the consumers, do not receive the payment directly.

where ϵ_p is the price elasticity of demand.

When $n > 1$, the expected profit for a firm producing $D(p_i)$ with price p_i can be written as

$$E\Pi(p_i, \theta_i) = Pr(p_i < p_{-i})p_i D(p_i) - \theta_i D(p_i), \text{ for all } i \in 1, 2..., n$$

For any given positive pricing strategy $p = \{p_i\}_{i=1}^n$, the winning probability, $Pr(p_i < p_{-i})$, is non-increasing in p_i under winner-take-all rule. As a result, $0 \leq p_i \leq p^m(\theta)$ and $\frac{\partial \pi_i(p_i,\theta_i)}{\partial p_i} \geq 0$ for all i. In this market, each firm wants to set a price as close as monopoly does, so as to maximize his profit given he wins. However, a high price induces a risk to lose the market. Thus a trade-off between cutting price to win the market and increasing price to maximize the profit occurs. Moreover, although production in advance implies that a firm with higher cost bears a higher unit loss no matter he wins or not, a negative sloping demand makes a high priced firm produces small amount of good, which in turn implies the total loss should be slight when the cost and quantity of demand are bounded. Next, we use the simple case with two firms and constant marginal cost to characterize the impact of the change in production process.

2.1 A simple example of production in advance

Now consider a simplified case where $n = 2$ and $c(q_i, \theta_i) = \theta_i q_i$ for $i \in \{1, 2\}$. Suppose each firm's cost is drawn from a common prior $\theta_i \in [\underline{\theta}, \overline{\theta}]$ with distribution $F(\theta_i)$.

The firm's expected profit in producing $D(p_i)$ with price p_i is

$$E\Pi(p_i, \theta_i) = Pr(p_i < p_j)p_i D(p_i) - \theta_i D(p_i).$$

Suppose a symmetric equilibrium pricing strategy $p^A(\theta_i)$, which is an increasing function of firm i's privately known cost θ_i. The winning probability $Pr(p^A(\theta_i) < p^A(\theta_{-i}))$ is therefore $1 - F(\theta_i)$. Also assume a boundary condition $p^A(\hat{\theta}) = \overline{p} < \infty$ such that $\pi(\overline{p}, \hat{\theta}) = 0$ for all $\theta \geq \hat{\theta}$. Rewrite the expected profit as

$$E\Pi(p^A(\theta_i), \theta_i) = (1 - F(\theta_i))p^A(\theta_i)D(p^A(\theta_i)) - \theta_i D(p^A(\theta_i)).$$

To argue the equilibrium, suppose firm 1 bids $p^A(z)$ instead of $p^A(\theta_1)$, and his profit is

$$E\Pi(p^A(z), \theta_1) = (1 - F(z))p^A(z)D(p^A(z)) - \theta_1 D(p^A(z)),$$

or

$$E\Pi(p^A(z), \theta_1) = \left[(1 - F(z))p^A(z) - \theta_1\right] D(p^A(z)). \tag{3}$$

First order condition for firm 1's profit maximization is

$$\begin{aligned} 0 =& \frac{\partial E\Pi(p^A(z), \theta_1)}{\partial z} \\ =& \left[(1 - F(z))p^A(z) - \theta_1\right] D'^A(z))p^{A'}(z) \\ &+ \left[-f(z)p^A(z) + (1 - F(z))p^{A'}(z)\right] D(p^A(z)) \end{aligned}$$

It can be rewrittern as

$$p^{A'}(z) = \frac{f(z)p^A(z)D(p^A(z))}{(1-F(z))(D(p^A(z)) + p^A(z)D'^A(z)) - \theta_1 D'^A(z))}$$

or

$$p^{A'}(z) = \frac{f(z)p^A D(p^A)}{(1-F(z))(D(p^A) + (p^A - \theta_1)D'^A) - F(z)\theta_1 D'^A)}. \quad (4)$$

Recall $\Gamma(1st)$, it is similar to first price auction. The expected profit function of using the symmetric equilibrium, $p^{1st}(z)$, in $\Gamma(1st)$ is

$$E\Pi(p^{1st}(z), \theta_1) = (1-F(z))(p^{1st}(z) - \theta_1)D(p^{1st}(z))$$

and subject to initial condition $p^{1st}(\bar{\theta}) = \bar{p}$. The first order condition can be written as

$$p^{1st'}(z) = \frac{f(z)(p^{1st} - \theta_1)D(p^{1st})}{(1-F(z))(D(p^{1st}) + (p^{1st} - \theta_1)D'^{1st}))}. \quad (5)$$

Constant demand If the demand function is constant, $D(p) = k$, the firms in game $\Gamma(A)$ set higher price than the firms in game $\Gamma(1st)$. To see this result, by (3),

$$0 = \frac{\partial E\Pi(p^A(z), \theta_1)}{\partial z} = \left[-f(z)p^A(z) + (1-F(z))p^{A'}(z)\right] k,$$

which implies, in equilibrium,

$$p^{A'}(\theta) = \frac{f(\theta)p^A(\theta)}{1-F(\theta)}.$$

The equilibrium pricing strategy can be solved as

$$p^A(\theta) = \frac{\overline{\theta}}{1 - F(\theta)}. \tag{6}$$

On the other hand by (5) and $D'(p) = 0$, we have

$$p^{1st'}(\theta) = \frac{f(\theta)(p^{1st}(\theta) - \theta)}{1 - F(\theta)}.$$

The equilibrium pricing strategy can be solved as

$$p^{1st}(\theta) = \frac{1}{1 - F(\theta)} \int_{\theta}^{\overline{\theta}} y f(y) dy. \tag{7}$$

Comparing (6) and (7), we have

$$p^A(\theta) > p^{1st}(\theta),\ \textit{for all}\ \underline{\theta} < \theta < \overline{\theta} \tag{8}$$

as demand is constant. This is a similar but reverse result on the buyers' auction that all-pay auction bidders bid lower than the bidders in first price auction. The difference is that players name a more aggressive in first-price mechanism than the players do in all-pay mechanism. We use this result as a linkage to compare p^A and p^{1st} in decreasing demand function. A quick result from Hansen [4] is the relation of equilibrium pricing between fixed demand and variable demand of game $\Gamma(1st)$:

$$p^{1st}(\theta)|_{D'=0} > p^{1st}(\theta)|_{D'<0}. \tag{9}$$

When θ is large, $0 < \overline{\theta} - \theta << 1$, the market demand shrinks to zero:

$$p^{A'}(\overline{\theta})|_{D'<0} = 0 < p^{A'}(\overline{\theta})|_{D'=0}$$

Moreover, by the boundary condition in both cases of market demand, $\overline{p} = p^A(\overline{\theta})|_{D'<0} = p^A(\overline{\theta})|_{D'=0}$, we have

$$p^A(\theta)|_{D'<0} > p^A(\theta)|_{D'=0},\ \textit{for all}\ 0 < \overline{\theta} - \theta << 1. \tag{10}$$

Summarizing (8), (9) and (10) gives us

$$p^A(\theta)|_{D'<0} > p^A(\theta)|_{D'=0} > p^{1st}(\theta)|_{D'=0} > p^{1st}(\theta)|_{D'<0}.$$

The lemma below has been proved.

Lemma 1 *If $0 < \overline{\theta} - \theta << 1$, the equilibrium pricing in $\Gamma(A)$, $p^A(\theta)$, is higher than it in $\Gamma(1st)$, $p^{1st}(\theta)$.*

Recall the expected profit functions of both game and rewrite them in equilibrium as

$$E\Pi^A(\theta) \equiv E\Pi(p^A(\theta), \theta) = (1 - F(\theta))(p^A(\theta) - \theta)D(p^A(\theta)) - F(\theta)\theta D(p^A(\theta)); \tag{11}$$

$$E\Pi^{1st}(\theta) \equiv E\Pi(p^{1st}(\theta), \theta) = (1 - F(\theta))(p^{1st}(\theta) - \theta)D(p^{1st}(\theta)). \tag{12}$$

Also define the difference in profit between two games as $\Delta(\theta) \equiv E\Pi(\theta)^{1st} - E\Pi^A(\theta)$. We want to show that $\Delta(\theta) \geq 0$ for all $\theta < \overline{\theta}$.

Lemma 2 *The expected profit of firms in game* $\Gamma(A)$ *is no larger than in* $\Gamma(1st)$.

Proof. Assume the negation. That is $\Delta(\theta) < 0$ for some firm with cost $\theta < \overline{\theta}$. Then by (11) and (12), $p^*(\theta) < p^m(\theta)$ implies $p^{1st}(\theta) < p^A(\theta)$. From the concavity of π, it follows $\pi'^{A}) > \pi'^{1st})$. Thus by Envelope theorem, $\frac{dE\Pi^A}{d\theta} > \frac{dE\Pi^{1st}}{d\theta}$, which means $\Delta'(\theta) < 0$. With the boundary condition that the largest cost firm earns zero expected profit in both games, $\Delta(\overline{\theta}) = 0$, it follows $\Delta(\theta) \geq 0$ for $\theta < \overline{\theta}$. ■

If $\theta = \underline{\theta}$, the winning probability is 1, and

$$\begin{aligned}\Delta(\underline{\theta}) &= \left[p^{1st}(\underline{\theta}) - \underline{\theta}\right] D(p^{1st}(\underline{\theta})) - \left[p^A(\underline{\theta}) - \underline{\theta}\right] D(p^A(\underline{\theta})) \\ &= \pi(p^{1st}(\underline{\theta})) - \pi(p^A(\underline{\theta})) \\ &\geq 0.\end{aligned}$$

Since the equilibrium price in both $\Gamma(A)$ and $\Gamma(1st)$ is smaller than p^m and p^m is the price such that $\pi' = 0$,

$$\pi'^g = p^{g'}(\underline{\theta}) D(p^{g'}(\underline{\theta})) + (p^g(\underline{\theta}) - \underline{\theta}) D(p^{g'}(\underline{\theta})) > 0$$

for both game $g \in \{A, 1st\}$, then

$$p^A(\theta) < p^{1st}(\theta), \text{ if } 0 < \theta - \underline{\theta} << 1. \tag{13}$$

Lemma 3 *If* $0 < \theta - \underline{\theta} << 1$, *the equilibrium pricing in* $\Gamma(A)$ *is lower than it in* $\Gamma(1st)$.

We summarize Lemma 1-3 in the following proposition.

Proposition 4 *Assume the market demand is decreasing in price. In game* $\Gamma(A)$*, the firms produce in advance, the equilibrium price with high cost* θ *is larger than the equilibrium price in game* $\Gamma(1st)$*; the equilibrium price,* $p^A(\theta)$*, with low cost* θ *is smaller than the equilibrium price,* $p^{1st}(\theta)$*.*

This is another perspective of seller's auction of Fibich et al.[3]. In Fibich et al.[3], the risk-averse buyer proposes a lower bid in all-pay auction to avoid a large loss with a relatively low winning probability. There is the similar effect in $\Gamma(A)$, as high cost firm charging a high price induces a relatively low demand, he can prevent loss in trade with low winning probability and, given winning, a low total revenue. On the other hand, the buyer with high value will bid high to avoid loss in the possibility of winning in Fibich et al. [3]. Here, the low cost firm deters his opponent with an aggressive pricing.

As a consequence, the proposition above implies when the equilibrium pricing functions are continuous, there is a realized cost θ^* such that $p^{1st}(\theta^*) = p^A(\theta^*) = p^*$, then $p^{A'}(\theta^*) > p^{1st'}(\theta^*)$ at θ^*. It follows that $p^A(\theta) \leq p^{1st}(\theta)$ for each firm whose cost is smaller than θ^*. By (4)-(5),

$$p^{A'}(\theta^*) > p^{1st'}(\theta^*),$$

which implies

$$1 - F(\theta^*) > -\frac{p^* - \theta^*}{D(p^*)} D'(p^*),$$

or

$$1 - F(\theta^*) > \frac{p^* - \theta^*}{p^*} \epsilon_{p^*}, \tag{14}$$

where ϵ_{p^*} is the price elasticity of demand evaluated at price p^*. The condition above means the probability to win is larger than the price mark-up ratio times the inverse of price elasticity of market demand. To illustrate the intuition, assume a constant elasticity of demand, $\epsilon_p = e > 0$. Although the winning bid is so large that the chance to win is small, as long as the mark-up or the elasticity of demand is large enough to compensate by taking all the market share. All-pay auction requires each firm to pay the cost to produce no matter the firm wins or not, and thus a more elastic market induces more incentive for firm to bid a low price. Indeed, this is the case when a firm draws a very low cost and offers a relatively low price, given the fact that a negative demand function is more elastic when the price is low.

Corollary 5 *When production starts before pricing, the equilibrium price is smaller than the one when production is after pricing if the market demand is relatively elastic.*

2.2 Pre-commitment capacity in Bertrand competition

Now, let's assume the duopoly case again. Suppose each firm knows the other firm's quantity produced, when he is setting his price, p_i. That is, firm 1 maximizes the following expected profit function given q_1, q_2

$$\max_{p_1} \; Pr(p_1 \leq p_2 | q_1, q_2) p_1 D(p_1) - \theta_1 q_1.$$

The information about firms' quantities provides a device of signalling that both firms will collude at the price that both firm sell out their product, given a proper belief system.

First, we show that under any quantity profile (q_1, q_2) revealed, coordinating on Cournot price $D^{-1}(q_1+q_2)$ is a Beysian Nash equilibrium at the pricing sub-game, no matter what belief system they constructed. For a given belief, the Nash equilibrium pricing is to bid either 0 or $D^{-1}(q_1+q_2)$. $p=0$ is an equilibrium since the pricing has no cost in this stage, and other price larger than zero induce zero demand. On the other hand, suppose both firm bid the Cournot price, $D^{-1}(q_1+q_2)$, in spite of a price cut, $p'^{-1}(q_1+q_2)$, induces larger demand and earns full market share, the sales still equals q_i and the profit is reduced. That is

$$D^{-1}(q_1+q_2)q_i - \theta_i q_i > p' q_i - \theta_i q_i, \; fori \in \{1,2\}.$$

Thus, the price $D^{-1}(q_1+q_2)$ is a Nash equilibrium. Since the type, i.e., the firm's cost, is irrelevant at this stage, any belief system will support this Nash equilibrium.

Next, consider the production stage. Defining the Cournot outcome for firm i, $q_i^c = q^c(\theta_i)$, is the solution of

$$\max_{q_i} \; D^{-1}(q_i + q_{-i})q_i - \theta_i q_i, fori \in \{1,2\}.$$

Given the best response of pricing stage, to produce q_i^c is the best response to the pricing subgame. This is because, for firm 1, to produce a quantity q' is no better than produce q_1^c given q_2^c. To see this, at the pricing equilibrium $D^{-1}(q_1+q_2)$,

$$D^{-1}(q_1^c + q_2^c)q_1^c - \theta_1 q_1^c > D^{-1}(q' + q_2^c)q' - \theta_1 q',$$

which is the definition above of q_i^c. All types of θ_i produce the quantity $q_i^c(\theta_i)$, the information is fully revealed at the second stage.

3 Concluding remarks

The seller's auction provides us a different view to investigate Bertrand competition. When firms should produce before they name the price, low cost firm will be more aggressive than the firm in a market that launching the production after naming the price. Moreover, a market with more elastic demand amplifies this effect.

When the firms is able to observe the quantity their rivals made, the outcome changes dramatically. The market equilibrium will be consistent with the Cournot outcome, though firms compete for market share as Bertrand game. This is a similar result to Kreps and Scheinkman[5] and Leopore [7]. The difference is the rationing rule for pricing competition. In the literature, they construct a complex rationing rule to deal with the problem of inventory. Here, we use the winner-take-all rule to simplify the model and prove a Cournot outcome in the Bertrand game settings with unknown rival's cost.

Finally, the produced in advance makes the firms not only gain lower profits than the ones in common Bertrand games, but also produce goods that cannot be sold in the winner-take-all market. Compensating with the larger demand which leads to more consumer surplus, the social welfare is undetermined when we consider the unsold goods induced from aggressive behavior of firms in competition. The solution seems to be that all parties are in favor of profits under circumstances would the firms reveal the quantities supplied before naming the price. Such recommendation is the outcome we have shown at the pre-commitment game.

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